Received: *September 8, 1999,* Accepted: *October 11, 1999*

Lattice matrix elements confronting the experimental value of ε'/ε

Stefano Bertolini and Marco Fabbrichesi

INFN, Sezione di Trieste and
Scuola Internazionale Superiore di Studi Avanzati (SISSA)
via Beirut 4, I-34013 Trieste, Italy
E-mail: stefano@he.sissa.it, marco@he.sissa.it

Abstract: A new lattice estimate of $K \to 2\pi$ transitions claims, contrary to all other computations, that the hadronic matrix element for the gluon penguin operator Q_6 has opposite sign and, in addition, is much larger than the vacuum saturation approximation. We comment under what conditions (if any) it is possible to reconcile this lattice result with the experimental value of ε'/ε. The dramatic impact of new physics in the kaon system that seems to be required is not easily accommodated within our present theoretical understanding.

Keywords: Kaon Physics, CP violation, Lattice QCD.

JHEP10(1999)011

A new lattice estimate of $K \to 2\pi$ transitions using domain-wall fermions claims [1], contrary to all other computations, that the hadronic matrix element for the gluon penguin operator Q_6 has opposite sign and, in addition, is much larger than its vacuum saturation approximation. This surprising result comes about because of the contribution of the so-called eye-contraction diagrams. The resulting value for the CP violating parameter ε'/ε is of the opposite sign and almost one order of magnitude bigger than the current experimental determination:

$$\varepsilon'/\varepsilon = (2.1 \pm 0.46) \times 10^{-3} \,, \tag{1}$$

which is obtained by averaging over the KTeV [2] and NA48 [3] preliminary results as well as the older 1992-93 experiments (E731 [4] and NA31 [5]).

If this lattice result (taken at its face value) will stand further scrutiny it raises serious questions on our understanding of electroweak physics in the kaon system.

In this letter we would like to discuss under what conditions (if any) it is possible to reconcile a large and positive value of $\langle 2\,\pi|Q_6|K\rangle$ with the experimental value of ε'/ε. We consider two possible scenarios:

- A modification of short-distance physics that changes both the sign and the size of the Wilson coefficient of the gluon or electroweak penguin operators. This can in principle be achieved by tampering with the initial matchings of the various coefficients while preserving the standard basis of operators. As we shall discuss, the behavior of the renormalization group equations (RGE) for the relevant effective lagrangian forces us to rather extreme changes in order to achieve the desired effect. Even though supersymmetry provides a framework for potentially large corrections, to construct a model in which such large deviations from standard physics are present in ε' while conspiring to be invisible everywhere else involves a contrived set of assumptions.

- An enlargement of the standard operator basis. We limit our discussion to the case of the chromomagnetic penguin operator because it does not affect the renormalization of the Wilson coefficients of the other dimension six operators (allowing us to draw model-independent conclusions) and it has been already shown to be a promising candidate for new-physics effects in ε'/ε. Given the current estimate of this operator's matrix element, short-distance changes alone—although potentially very large—are not sufficient in bringing ε'/ε in the experimental ball-park. A final assessment requires a lattice evaluation of the hadronic matrix element of the chromomagnetic operator.

Both scenarios call for a dramatic impact of new physics in the kaon system and they are not easily accommodated within our present theoretical understanding. More exotic extensions of the standard operator basis and more extreme scenarios require a detailed model-dependent discussion which is beyond the scope of the present letter.[1]

Let us fix our notation by introducing the $\Delta S = 1$ effective four-quark lagrangian for $m_c < \mu < m_b$

$$\mathcal{L}_{eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}}\lambda_u\Big\{(1-\tau)\sum_{i=1,2} C_i(\mu)[Q_i(\mu) - Q_i^c(\mu)] + \tau \sum_{i=1,10} C_i(\mu)Q_i(\mu)\Big\}\,, \qquad (2)$$

where $\lambda_q \equiv V_{qd}V_{qs}^*$ and $\tau = -\lambda_t/\lambda_u$. In the discussion that follows we will assume as the standard-model reference value for the CP phase $\mathrm{Im}\,\lambda_t = 10^{-4}$. The renormalization group Wilson coefficient $C_i(\mu)$ are known to the next-to-leading order in α_s and α_e [7, 8].

The standard basis of effective operators is

$$\begin{aligned}
Q_1 &= (\overline{s}_\alpha u_\beta)_{\mathrm{V-A}}\,(\overline{u}_\beta d_\alpha)_{\mathrm{V-A}}\,, & Q_1^c &= (\overline{s}_\alpha c_\beta)_{\mathrm{V-A}}\,(\overline{c}_\beta d_\alpha)_{\mathrm{V-A}}\,,\\
Q_2 &= (\overline{s}u)_{\mathrm{V-A}}\,(\overline{u}d)_{\mathrm{V-A}}\,, & Q_2^c &= (\overline{s}c)_{\mathrm{V-A}}\,(\overline{c}d)_{\mathrm{V-A}}\,,\\
Q_{3,5} &= (\overline{s}d)_{\mathrm{V-A}}\textstyle\sum_q (\overline{q}q)_{\mathrm{V\mp A}}\,, &&\\
Q_{4,6} &= (\overline{s}_\alpha d_\beta)_{\mathrm{V-A}}\textstyle\sum_q (\overline{q}_\beta q_\alpha)_{\mathrm{V\mp A}}\,, &&\\
Q_{7,9} &= \frac{3}{2}\,(\overline{s}d)_{\mathrm{V-A}}\sum_q \hat{e}_q\,(\overline{q}q)_{\mathrm{V\pm A}}\,, &&\\
Q_{8,10} &= \frac{3}{2}\,(\overline{s}_\alpha d_\beta)_{\mathrm{V-A}}\sum_q \hat{e}_q(\overline{q}_\beta q_\alpha)_{\mathrm{V\pm A}}\,, &&
\end{aligned} \qquad (3)$$

where $V \pm A$ stands for $\gamma_\mu(1 \pm \gamma_5)$ and $\hat{e}_q$ is the value of the electric charge of the quark $q = u, d, s, c$.

Taking the hadronic matrix elements of the operators in eq. (3) obtained by the lattice calculation [1], and given their standard-model Wilson coefficients, ε'/ε is completely dominated by the contribution of Q_6 and is therefore large and negative (in dramatic disagreement with the experiment).

In order to reconcile the lattice and the experimental results, we must somehow compensate for this dominant contribution. The ratio ε'/ε is determined by the sum of isospin $I = 0$ and $I = 2$ amplitudes. Let us examine in turns possible modifications on these two classes of contributions.

A first possibility, in the $I = 0$ amplitude, is that the Wilson coefficient of the Q_6, at the scale at which the matrix element is computed (about $2\,\mathrm{GeV}$), is changed with respect to its standard-model value in order to reproduce the experimental result. Assuming that new physics only modifies the initial conditions (at m_W) of the RGE, we need to know how these changes are propagated by the RGE down to the hadronic scale. This has been discussed for the whole operator basis in section VII of ref. [9].

[1] Possible effects of dissipation and loss of coherence in the kaon system relevant to ε'/ε have been discussed in ref. [6].

In order to keep the discussion as model independent as possible, we parameterize all deviations from the standard-model matching conditions $C_i(m_W)$ in terms of the parameters

$$r_i \equiv C_i(m_W)/C_i^{SM}(m_W)\,. \tag{4}$$

In general, a Wilson coefficient at the low scale receive contributions from both a multiplicative and additive renormalization, the latter arising from QCD-induced operator mixing. In particular, the $C_6(\mu)$ is dominated in the standard basis by the additive renormalization induced by the mixing of Q_6 with the Q_2 operator.

We have plotted in fig. 1 the Wilson coefficient $C_6(2\,\text{GeV})$ as a function of the parameters r_2 and r_6. The dark (red) band represents the values of C_6 for which the experimental result for ε'/ε would be recovered (leaving all other Wilson coefficients unchanged) that is

$$-0.2 < \frac{C_6(2\text{ GeV})}{C_6^{SM}(2\text{ GeV})} < -0.1\,. \tag{5}$$

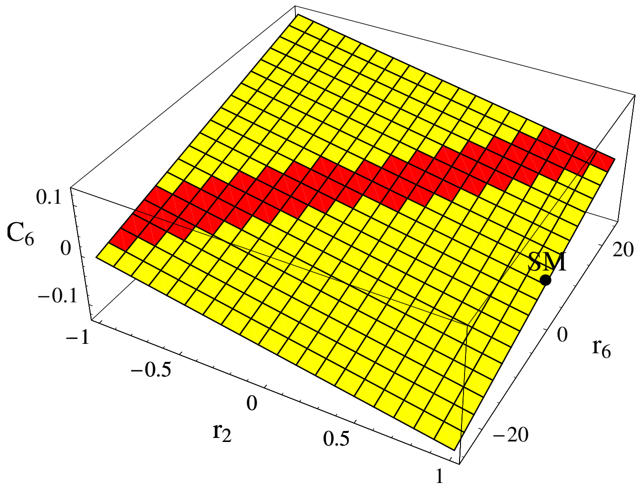

Figure 1: The Wilson coefficient $C_6(2\,\text{GeV})$ as a function of the parameters r_2 and r_6 defined in eq. (4).

As it can seen by inspection, this possibility is realized either by a drastic reduction of $C_2(m_W)$ (the left side of fig. 1) or a large enhancement of $C_6(m_W)$ (the right side of fig. 1). However, the value of $C_2(2\,\text{GeV})$ is severely constrained by the CP-conserving amplitude $A_0(K \to 2\pi)$, which in the same lattice calculation is reproduced up to a factor of two. Moreover, it is difficult to substantially change $C_2(m_W)$ by means of new physics because this Wilson coefficient is due to tree level W-exchange.

Looking then at fig. 1, and given that $C_2(m_W)$ cannot differ too much from its standard-model value, it is possible to reproduce the experimental ε'/ε only by enhancing $C_6(m_W)$ by more than a factor of twenty. However, such a large enhancement can hardly take place without affecting other processes and we shall come back to it after discussing the electroweak penguin.

Leaving the $I = 0$ contribution alone, we can still compensate for the large and negative result by acting on the the $I = 2$ contribution. Here the dominant operator is Q_8. Given the lattice estimate of the matrix element of this operator, agreement with the experiments would require

$$C_8(2\,\text{GeV}) \simeq -30\;\; C_8^{SM}(2\,\text{GeV})\,. \tag{6}$$

$\mu = m_W$	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Tree		$\surd$								
Tree + g	$\surd$	$\surd$								
Tree + γ		$\surd$								
$\mathcal{P}_g$			$\surd$	$\surd$	$\surd$	$\surd$				
$\mathcal{P}_\gamma$							$\surd$		$\surd$	
$\mathcal{P}_Z$			$\surd$				$\surd$		$\surd$	
$\mathcal{B}$			$\surd$						$\surd$	

Table 1: Contributions to the one-loop matching of the $\Delta S = 1$ Wilson coefficients at $\mu = m_W$. Non-vanishing contributions to C_8 and C_{10} arise via the QCD renormalization of the operators Q_7 and Q_9, respectively. The label "Tree" stands for tree-level W exchange, $\mathcal{P}_{g,\gamma,Z}$ for gluon, photon and Z-induced penguins, while $\mathcal{B}$ for the W-induced box diagrams.

The RGE analysis shows that $C_8(2\,\text{GeV})$ varies proportionally to $C_7(m_W)$ (recall that $C_8(m_W)$=0) and, as one can see from table 1, $C_7(m_W)$ receives contributions from photon and Z penguins.

To date, the best limits on the CP conserving component of the Z-penguin operator, $\mathrm{Re}\,\mathcal{P}_Z$, are provided by the $K_L \to \mu^+\mu^-$ decay [10, 11, 12], whose branching ratio is measured to be [13]

$$B(K_L \to \mu^+\mu^-) = (7.18 \pm 0.17) \times 10^{-9} \;, \tag{7}$$

even though the constraint on $\mathrm{Re}\,\mathcal{P}_Z$ is not as accurate as the experimental precision because of the theoretical long-distance uncertainties related to the two-photon component [14].

The cleanest constraint on $|\mathcal{P}_Z|$ comes from the decay $K^+ \to \pi^+\bar{\nu}\nu$, which is currently measured to be [15]

$$B(K^+ \to \pi^+\bar{\nu}\nu) = 4.2\ ^{+9.7}_{-3.5} \times 10^{-10} \;. \tag{8}$$

Due to the lack of further evidence, the branching ratio in eq. (8) is going to be reduced by a factor of two or three [16].

Taking the standard-model expectation $B(K^+ \to \pi^+\bar{\nu}\nu) = (0.8{\pm}0.3){\times}10^{-10}$ [17] as a reference point, a numerical analysis shows that we can at most modify $\mathrm{Im}\,\mathcal{P}_Z^{SM}$ by a factor of sixteen.

Barring the unlikely possibility of independent and widely different effects in the effective $(\bar{s}d)_{V-A}$ vertex of the photon and Z penguins, eq. (8) rules out the enhancement required by eq. (6). Analogous considerations hold for the other electroweak operators which would entail even larger deviations of their Wilson coefficients from the standard-model values. By the same token, also the enhancement by a factor of twenty of the gluonic penguin coefficient $C_6(m_W)$ — which we argued is necessary in the $I = 0$ amplitude in order to reproduce the experimental value of ε'/ε — is difficult to accommodate.

JHEP10(1999)011

A notable exception to the above argument is the presence of gluino-induced flavour-changing neutral currents in supersymmetric models. The gluino induced $\Delta S = 1$ transitions are suppressed in Z-penguin diagrams compared to gluon or photon penguins by a factor of $O(m_K^2/m_Z^2)$ (for a detailed discussion of this effect which follows from gauge invariance see ref. [18]). As a consequence, the bounds on Z-penguins are not effective on gluino-induced gluon and photon penguins. However, the gluino contributions to the standard $\Delta S = 1$ penguin operators are indirectly constrained by the sharp bounds on the gluino-induced $\Delta S = 2$ box diagrams relevant to $\bar{K}^0 - K^0$ transitions [19].

Gluino-box diagrams can also play a direct role in $\Delta S = 1$ amplitudes. In ref. [20], it is shown that gluino-box diagrams may induce a potentially large isospin-breaking contribution to the electroweak penguin Q_8 while satisfying all other bounds. However, the large factor required by eq. (6) implies a rather large mass splitting between the right-handed squark isospin doublet ($m_{\tilde{u}_R} - m_{\tilde{d}_R} \simeq 1\,\mathrm{TeV}$) together with a large gluino mass in order to evade the bounds from Δm_K and ε.

An operator not usually included in the standard-model analysis of ε'/ε is the chromomagnetic penguin

$$Q_{11} = \frac{g_s}{8\pi^2}\, \bar{s}\, \left[m_d R + m_s L\right]\, \sigma_{\mu\nu}\, G^{\mu\nu}\, d\,, \tag{9}$$

where $R(L) = (1 \pm \gamma_5)/2$ and $G^{\mu\nu}$ is the gluon field.

The matrix element $\langle 2\pi|Q_{11}|K\rangle$ has been computed in the chiral quark model and shown to arise only at $O(p^4)$ in the chiral expansion and to be further suppressed by a m_π^2/m_K^2 factor with respect to the naive expectation [21]. For these reasons, even though the Q_{11} Wilson coefficient receives a large additive renormalization, its standard-model contribution to ε'/ε has been shown to be negligible [21].

This is no longer true if the Cabibbo-Kobayashi-Maskawa (CKM) suppression of its CP violating component can be lifted without violating other bounds. A clever example of it is discussed in ref. [22], where the standard-model factor $\mathrm{Im}\,\lambda_t \simeq O(\sin^5\theta_C)$ is replaced in a supersymmetric framework by a CP violating squark mixing of $O(\sin\theta_C)$, thus introducing a potential enhancement of the chromomagnetic Wilson coefficient by three orders of magnitude. This solution allows for a contribution to ε'/ε at the 10^{-3} level by saturating the known bounds coming from CP violating phenomenology (discussions of the implications of these bounds on various supersymmetric models can be found in ref. [23]).

This enhancement of C_{11} is still not enough to compensate for the huge negative Q_6 contribution to ε'/ε of the lattice result; in fact, keeping fixed the gluon penguin contribution, agreement with experiment would require a matrix element for Q_{11} larger by about a factor of ten. Actually, the leading $O(p^4)$ chiral quark model estimate of the hadronic matrix element may receive potentially large $O(p^6)$ contributions if the accidental m_π^2/m_K^2 suppression is replaced by m_K^2/Λ_χ^2. To further assess this possibility, it would therefore be interesting to have an estimate of the Q_{11} matrix element from the same lattice approach that produces the large and positive matrix element for the Q_6.

Clearly, extensions of the standard model can also involve new effective operators beyond the standard basis of eq. (3) and Q_{11}. However, in this case very little can be said without a complete re-analysis of ε'/ε. Similarly, scenarios in which the CKM matrix is taken to be real and CP violation arises only in the new-physics sector can in principle be invoked but again require a detailed model-dependent analysis before being considered a viable alternative.

In conclusion, while a combination of the above scenarios may make some of the requirements less severe, a viable model which avoids the phenomenological bounds discussed is forced to rely on a contrived choice of parameters. It is nevertheless remarkable how supersymmetry can provide a framework for potentially large effects on ε'/ε while satisfying all present data.

It is also fair to add that the lattice result by means of domain-wall fermions in ref. [1] must stand further scrutiny and corroboration before concluding that the standard model is facing its most dramatic challenge to date.

Acknowledgments

We thank F. Borzumati and M. Neubert for useful discussions and comments.

References

[1] RIKEN-BNL-Columbia collaboration, *A first study of ε'/ε on the lattice using domain wall fermions*, hep-lat/9908025.

[2] KTeV collaboration, *Phys. Rev. Lett.* **83** (1999) 22 [hep-ex/9905060].

[3] P. Debu, seminar at CERN, June 18, 1999 (http://www.cern.ch/NA48).

[4] E731 collaboration, *Phys. Rev.* **D 55** (1997) 6625.

[5] NA31 Collaboration, *Phys. Lett.* **B 317** (1993) 233.

[6] F. Benatti and R. Floreanini, *Mod. Phys. Lett.* **A 14** (1999) 1519 [hep-ph/9906272].

[7] A.J. Buras, M. Jamin, E. Lautenbacher and P.H. Weisz, *Nucl. Phys.* **B 370** (1992) 69; *Nucl. Phys.* **B 400** (1993) 37 [hep-ph/9211304];
A.J. Buras, M. Jamin and M.E. Lautenbacher, *Nucl. Phys.* **B 400** (1993) 75 [hep-ph/9211321]; *Nucl. Phys.* **B 408** (1993) 209 [hep-ph/9303284].

[8] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *Nucl. Phys.* **B 415** (1994) 403 [hep-ph/9304257].

JHEP10(1999)011

[9] S. Bertolini, J.O. Eeg and M. Fabbrichesi, *Estimating ε'/ε. A review*, hep-ph/9802405, to appear on *Rev. Mod. Phys.*

[10] Y. Grossman and Y. Nir, *Phys. Lett.* **B 398** (1997) 163 [hep-ph/9701313].

[11] G. Colangelo and G. Isidori, *J. High Energy Phys.* **09** (1998) 009 [hep-ph/9808487].

[12] A.J. Buras and L. Silvestrini, *Nucl. Phys.* **B 546** (1999) 299 [hep-ph/9811471];
A.J. Buras, G. Colangelo, G. Isidori, A. Romanino and L. Silvestrini, *Connections between ε'/ε and rare kaon decays in supersymmetry*, hep-ph/9908371.

[13] D. Ambrose, talk presented at the Kaon 99 Conference, University of Chicago, June 1999.

[14] G. D'Ambrosio, G. Isidori and J. Portoles, *Phys. Lett.* **B 423** (1998) 385 [hep-ph/9708326].

[15] E787 collaboration, *Phys. Rev. Lett.* **79** (1997) 2204 [hep-ex/9708031].

[16] G. Redingler, talk presented at the Kaon 99 Conference, University of Chicago, June 1999.

[17] G. Buchalla and A.J. Buras, *Phys. Rev.* **D 54** (1996) 6782 [hep-ph/9607447].

[18] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, *Nucl. Phys.* **B 353** (1991) 591.

[19] E. Gabrielli and G.F. Giudice, *Nucl. Phys.* **B 433** (1995) 3 [hep-ph/9407029];
F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *Nucl. Phys.* **B 477** (1996) 321 [hep-ph/9604387].

[20] A.I. Kagan and M. Neubert, hep-ph/9908404.

[21] S. Bertolini, E. Gabrielli and M. Fabbrichesi, *Phys. Lett.* **B 327** (1994) 136 [hep-ph/9312266];
S. Bertolini, J.O. Eeg and M. Fabbrichesi, *Nucl. Phys.* **B 449** (1995) 197 [hep-ph/9409437].

[22] A. Masiero and H. Murayama, *Phys. Rev. Lett.* **83** (1999) 907 [hep-ph/9903363].

[23] K.S. Babu, B. Dutta and R.N. Mohapatra, *Seesaw-constrained MSSM, solution to the SUSY CP problem and a supersymmetric explanation of ε'/ε*, hep-ph/9905464;
S. Khalil and T. Kobayashi, *Phys. Lett.* **B 460** (1999) 341 [hep-ph/9906374];
S. Baek et al., *Fully supersymmetric CP violations in the kaon system*, hep-ph/9907572;
R. Barbieri, R. Contino and A. Strumia, *ε' from supersymmetry with non universal A terms?*, hep-ph/9908255;
A.J. Buras et al., *Connections between ε'/ε and rare kaon decays in supersymmetry*, hep-ph/9908371;
G. Eyal et al., *Probing supersymmetric flavor models with ε'/ε*, hep-ph/9908382.